RW metric

$$ds^2 = -c^2dt^2 + R(t)^2\left[d\chi^2 + S_k^2(\chi)d\psi^2\right]$$

$S_k(\chi) = \sin\chi, \chi, \sinh\chi$ [for closed (k=+1), flat (k=0), open (k=-1)]

$$d\psi^2 = d\theta^2 + \sin^2\vartheta\, d\phi^2$$

χ comoving radial coordinate [dimensionless]. Time independent.

R(t) scale factor. [length]

$a(t) = \dfrac{R(t)}{R(t_0)}$ [dimensionless]

Time

t **proper time** of a comoving observer.
-> cosmic time (in an isotropic universe)

conformal time

$$d\tau = dt / R(t)$$

(brings R(t) to front of metric

$$ds^2 = R(t)^2\left[-c^2d\tau^2 + d\chi^2 + S_k^2(\chi)d\psi^2\right]$$

$$\tau = \int_0^t \frac{dt'}{R(t')}$$

Distance

Proper distance

$$D(t) = R(t)\chi$$

This is the distance (along a constant time surface, dt=0) between us and a galaxy with comoving coordinate χ,
the distance a series of comoving observers would measure if they each lay their rulers end-to-end at the same cosmic instant.

Comoving distance (take out the Hubble flow).

Past light cone

Traces events in the Universe that we can currently see.
Photons travels along null geodesic ds=0.
The comoving coordinate of a comoving object that emitted the light we now see at time t is $\chi_{lc}(t_{em}) = c\int_{t_{em}}^{t_0} \frac{dt'}{R(t')}$

Redshift

Redshift given by scale factor at time of emission: $1 + z = \dfrac{R_0}{R(t)}$

-> relate comoving distance to redshift

$$\chi(z) = \frac{c}{R_0}\int_0^z \frac{dz'}{H(z')}$$

(n.b. redshift gives position, not velocity)

Redshift of an object is not constant: change over a time interval (t_0) in accelerating/decelerating universe:

$$\frac{dz}{dt_0} = H_0(1+z) - H_{em}$$

$$H_{em} = \dot{R}_{em}/R_{em}$$

very small effect. (fig).

Recession velocity

Total velocity is derivative of proper distance with respect to proper time,

$$v_{tot} = \dot{D}$$

$$\dot{D} = \dot{R}\chi + R\dot{\chi}$$

$$v_{tot} = v_{rec} + v_{pec}$$

v_{pec} is measured with respect to comoving observers coincident with the object in question. Must be less than or equal to c

v_{rec} is the velocity of the Hubble flow at proper distance D
can be arbitrarily large.

v_{rec} is given by the slope on the spacetime diagram.

Hubble law

$$v_{rec} = H(t)D = \dot{R}\chi$$

Derived directly from the metric. Valid for all distances in any homogeneous, expanding Universe.

n.b. Hubble's redshift-distance law

$$z = \frac{H_0}{c}D$$

empirical law, valid for small distances. (z << 1).

Hubble Sphere

Surface on which comoving objects are receding at the speed of light.
Sphere with radius D=c/H.

Misconceptions

#1: Recession velocities cannot exceed the speed of light

$$GR \quad v_{rec}(t,z) = \frac{c}{R_0}\dot{R}(t)\int_0^z \frac{dz'}{H(z')}$$

$$SR \quad v_{pec}(z) = c\,\frac{(1+z)^2 - 1}{(1+z)^2 + 1}$$

All objects with redshift greater than z~1.46 are receding faster than the speed of light.

#2: Inflation results in superluminal expansion but the normal expansion of the universe does not

#3: Galaxies with recession velocities exceeding the speed of light exist but we cannot see them.

Particle Horizon

Distance light can have traveled from *t=0* to a given time *t*.

$$\chi_{ph}(t) = c\int_0^t \frac{dt'}{R(t')}$$

separates comoving distances (particles) we can currently see from those we cannot currently see. At any particular time, the particle horizon forms a sphere around us beyond which we cannot *yet* see.

$R_0\chi_0 \sim 47\text{Glyr}$ is the most distant comoving object the we can currently observe. This is the size of the observable universe (objects that emitted those photons in the early Universe have moved that far away as the Universe expanded.)

Ambiguity: the depiction of particle horizons on space-time diagrams.
This dotted line is not a world-line. It shows the particle horizon at time *t*.

Event Horizon

Distance light can travel from a given time *t* to $t = \infty$

$$\chi_c(t) = c\int_t^{t_{end}} \frac{dt'}{R(t')}$$

separates events we are able to see at some time, from events we will never be able to see.
In $(\Omega_M, \Omega_\Lambda) = (0.3, 0.7)$ cosmology, galaxies we currently observe at $z \sim 1.8$ are just passing over our event horizon. Thus these galaxies are the most distant objects from which we will *ever* received information about the present day.

We will *never* see any objects at any time from beyond the maximum comoving distance of the particle and event horizons. (~62 Glyr).

Conclusions

All objects with redshift greater than z~1.46 are receding faster than the speed of light. Obviously, we can see many galaxies that are moving superluminaly.

This is the size of the currently observable Universe: $R_0\chi_0 \sim 47\,\mathrm{Glyr}$.

Galaxies we currently observe at $z \sim 1.8$ are just passing over our event horizon. Thus these galaxies are the most distant objects from which we will ever received information about the present day.

We will never see any objects at any time from beyond the maximum comoving distance of the particle and event horizons. (~62 Glyr).

Redshifts do not relate to velocities according to SR expectations.

Redshift of an object changes due to acceleration of universe.